Decision-theoretic modeling of early life failures in semiconductor manufacturing

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Outline

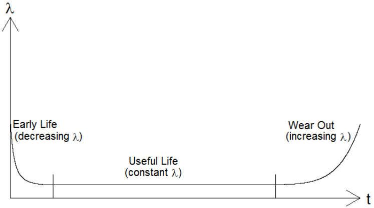
1. Introduction

Key issue in semiconductor manufacturing:

Reliability

most commonly applied failure screening technique: **Burn-in-study**, especially in safety-critical applications

Basis: bathtub curve describing hazard rate



Testing under accelerated stress conditions
(increased temperature & voltage stress)

Burn-in: independently selected number of devices is investigated for early failures

Model for early failures: **Weibull** distribution $Wb(a, b), b < 1$.

Current ppm-requirement: 21ppm (Infineon Technologies Villach, Austria)

Burn-in schemes different for logic and power devices. Here we focus on **power devices.**

Reasons for early failure: oxide particles, metallization defects,...

Problem: only very few failures

$\Rightarrow$ it's rarely possible to efficiently fit a Weibull DFR distribution to burn-in data.

Way out: prove that early life failure probability $p \in$ target confidence area

Burn-in read-outs at discrete time points t_1, t_2, t_3

Report statistics: $k_j = \#$ failures in $(t_{j-1}, t_j]$

$$j = 1, 2, 3;\ t_0 = 0$$

Goal: P (early life failure after t_3 hours) $\leq 21ppm$

Successful burn-in: requires $k = k_1 + k_2 + k_3 = 0$

(zero defect strategy)

Usually: Burn-in is re-started whenever a failure occurs

Current standard: introduction of **countermeasures** (CM)
(ink out, design measures, optical inspection, ...)
to reduce the failure probability p

Our aim:

- development of a statistical model for taking account of CM's
- avoid re-start of burn-in by planning additional number of items to be burnt for zero defects.

2. Interval estimation for early life failure probabilities

n independently selected devices are stressed

$$X_i = \begin{cases} 0 & \text{if device } i \text{ passes the burn-in} \\ 1 & \text{if device } i \text{ fails within burn-in} \end{cases}$$

$$X = \sum_{i=1}^{n} X_i \sim Bi(n, p)$$

$$\boldsymbol{x} = (x_1, \ldots, x_n) \in \{0,1\}^n; \quad k = \boldsymbol{x}^T \boldsymbol{x} \in \{0, 1, \ldots, n\}$$
$$= \#\text{ failures}$$

2.1 Clopper-Pearson interval estimation

$$I_{CP} = (\hat{p}_l, \hat{p}_u) \qquad \text{where}$$

$$P(X \geq k | \hat{p}_l) = \alpha/2 \quad \text{and}$$

$$P(X \leq k | \hat{p}_u) = \alpha/2$$

To obtain $\hat{p}_l$ and $\hat{p}_u$, we use the well-known relationship with the Beta distribution

$\hat{p}_l = F_{Z_l}^{-1}(\alpha/2)$ with $Z_l \sim Be(k, n-k+1)$

$\hat{p}_u = F_{Z_u}^{-1}(1-\alpha/2)$ with $Z_u \sim Be(k+1, n-k)$

90% one-sided interval $I_p = [0, \hat{p}_u]$; $\alpha/2 = 0.1$

2.2 Bayesian equal-tail interval for p

In a Bayesian framework, this relationship comes in naturally observing that the conjugate prior for p is the Beta distribution:

$$p \sim Be(a, b); \quad a, b > 0$$

$$\Rightarrow f(p|\boldsymbol{x}) \propto l(p; \boldsymbol{x}) f(p) = p^{a+k-1}(1-p)^{b+n-k-1}$$

$$\text{i.e. } p|\boldsymbol{x} \sim Be(a^* = a + k, b^* = b + n - k)$$

Bayesian equal-tail credible interval

$$C_e = (\hat{p}_l^*, \hat{p}_u^*) \text{ where } \hat{p}_l^* = F_{p|\boldsymbol{x}}^{-1}(\alpha/2), \hat{p}_u^* = F_{p|\boldsymbol{x}}^{-1}(1 - \alpha/2)$$

Jeffreys' prior: $a = b = 1/2$

Choosing $a = 1, b = 0$ we have

$$p|\boldsymbol{x} = Be(k + 1, n - k)$$

$$\hat{p}_u^* = \hat{p}_u$$

concidence of one-sided Bayesian interval with Clopper-Pearson interval

3. Assessing *ppm*-levels using CM's

Repair is impossible for semiconductor devices; they either pass or fail within the burn-in.

If a burn-in related failure occurs, then a CM is introduced (optical inspection, process improvement, ...) aiming to reduce *p* to $\pi \leq p$.

Crucial: Experts assess the CM's effectiveness $\vartheta \in [0, 1]$

$$\vartheta = \text{probability of correcting the failure.}$$

3.1 Single CM failure probability model

Consider k failures for which a single CM with effectiveness $\vartheta \in [0, 1]$ is implemented in the process

Interpretation: There is a likelihood ξ_j that $j \leq k$ failures would have occured or, equivalently, $k - j$ failures would have been corrected if the CM would have already been introduced before the burn-in study.

Let $K_l = \begin{cases} 1 & \text{if failure } l \text{ is corrected} \\ 0 & \text{else} \end{cases}$

Clearly: $K = \sum_{l=1}^{k} K_l \sim Bi(k, \vartheta)$

$\downarrow$

unknown number of failures that would have been caught by the CM

$\Rightarrow (*) \;\; \xi_j = P(K = k - j); \;\; j \in \{0, \ldots, k\}$

Clopper-Pearson model for single CM

after the CM: $X' \sim Bi(n, \pi)$

Weighting of Clopper-Pearson upper limits according to $(*)$ leads to assessing $\hat{\pi}$ as

$$\sum_{j=0}^{k} \xi_j P(X' \leq j | \hat{\pi}) = \alpha$$

Equivalently: using $P(X' \leq j|\pi) = 1 - P(Z_j < \pi)$

$$\text{with } Z_j \sim Be(j+1, n-j); \quad j = 0, \dots, k$$

$\Rightarrow \hat{\pi} = F_{Z'}^{-1}(1-\alpha) = (1-\alpha)$-quantile of

$$Z' \sim \sum_{j=0}^{k} \xi_j Be(j+1, n-j) \text{ Beta mixture}$$

Bayesian model for single CM

prior $\pi \sim Be(a, b)$

actual number of failures after CM introduction is $k - K$ and is unknown

Therefore consider the preposterior:

$$\Xi := E[\pi | k - K] = \sum_{j=0}^{k} \xi_j (\pi | j) \sim \sum_{j=0}^{k} \xi_j Be(a + j, b + n - j)]$$

$\rightarrow \hat{\pi}^* = F_{\Xi}^{-1}(1 - \alpha)$ = $(1 - \alpha)$ -quantile of the mixture distribution Ξ.

Again: $\hat{\pi}^* = \hat{\pi}$ for the prior $\pi \sim Be(1, 0)$

Setting $\vartheta = 0$ (no CM is implemented) we arrive at the classical estimation models.

3.2 Multiple CM failure model

now consider $r \geq 1$ different CM's and denote $\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_r) =$ vector of effectivenesses; $r \leq k$

$\boldsymbol{k} = (k_1, \ldots, k_r)$; $k_i = \#$ failures tackled by CM$_i$

$$\text{with } \sum_{i=1}^{r} k_i = k$$

Now: $K = \sum_{l=1}^{k} K_l \sim GBi(k, \boldsymbol{\vartheta}_k)$ **generalized binomial**, where

$$\boldsymbol{\vartheta}_k = (\underbrace{\vartheta_1, \ldots, \vartheta_1}_{k_1 \text{ times}}, \underbrace{\vartheta_2, \ldots, \vartheta_2}_{k_2 \text{ times}}, \ldots, \underbrace{\vartheta_r, \ldots, \vartheta_r}_{k_r \text{ times}})$$

We have developed an **efficient method** for computing generalized binomial probabilities employing **sequential convolution.**

3.3 CM's with uncertain effectivenesses

So far: $\vartheta_i, i = 1, \ldots, r$; were fixed

often: process experts are uncertain about the effectivenesses of the applied CM's.

a) Beta-Binomial model for a single uncertain effectiveness, $r = 1$

$$\vartheta \sim Be(u, v)$$

$$K|\vartheta \sim Bi(k, \vartheta)$$

$$\begin{aligned} \Rightarrow \xi_j \quad &= P(K = k - j) = \int_0^1 P(K = k - j|\vartheta) f(\vartheta) d\vartheta \\ &= \begin{pmatrix} k \\ k - j \end{pmatrix} \frac{\Gamma(u+k-j)\Gamma(v+j)}{\Gamma(u+k+v)} \; \frac{\Gamma(u+v)}{\Gamma(u)\Gamma(v)} \end{aligned}$$

$K \sim BeBi(k, u, v)$ Beta-Binomial

b) Generalized Beta-Binomial model for more than a single uncertain effectiveness

$$K|\vartheta \sim GBi(k, \vartheta_k)$$

$$\vartheta_i \sim Be(u_i, v_i); \quad i = 1, \ldots, r$$

$$\Rightarrow P(K = k - j) = \int_{[0,1]^r} P(K = k - j|\vartheta) f(\vartheta) d\vartheta$$

$$K \sim GBeBi(k, u_1, \ldots, u_r, v_1, \ldots, v_r, k_1, \ldots, k_r)$$

no closed form solution available,

MC-integration

4. Decision-theoretical formulation of the CM failure probability model

Parameter space: $p \in \Theta = [0, 1]$

after implementing CM's: $\pi \in \Theta' = \Theta = [0, 1]$ with $\pi \leq p$

Action space: without CM's $a = \hat{p} \in \mathcal{A} = [0, 1]$

after incorporating CM's: $a' = \hat{\pi} \in \mathcal{A}' = \mathcal{A} = [0, 1]$

Sample space of Burn-in data: $\boldsymbol{X}|p \sim Bi(n, p)$

$\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathcal{X} = \{0, 1\}^n$

can be sufficiently described by

$\mathcal{T} = \{\boldsymbol{x}^T \boldsymbol{x} : \boldsymbol{x} \in \mathcal{X}\} = \{0, 1, \ldots, n\}$

after implementing CM's: we **simulate** failure **scenarios** $j \in \mathcal{T}$, based on the observed $k \in \mathcal{T}; \;\; 0 \leq j \leq k$; as outcomes, which would have possibly occured if we would have introduced the CM's already before the burn-in.

To these scenarios we attach prob's ξ_j
(assessed wrt. the CM's effectivenesses)

Assessment of the ξ_j: for single CM by means of $Bi(k, \vartheta)$, i.e. simulation depends on $k \in \mathcal{T}$ and $\vartheta \in [0, 1]$.

in case of $r \leq k$ different CM's:

ξ_j determined by $GBi(k, \vartheta_{\boldsymbol{k}})$ where $\boldsymbol{k} = (k_1, \ldots, k_r) \in \mathcal{K}$ reports the number of failures k_j tackled by CM_i; $\;i = 1, \ldots, r$. There are

$$|\mathcal{K}| = \begin{pmatrix} r+k-1 \\ k \end{pmatrix} \text{ different vectors } \boldsymbol{k}$$

Simulations depend on observed $k \in \mathcal{T}, \vartheta \in [0,1]^r$ and $\boldsymbol{k}$

Decision functions

$d : \mathcal{T} \to A$ and $d(k) = \hat{p}$ $ppm-$level estimator extension in the CM decision framework

Single CM case: $d' : \mathcal{T} \times [0,1] \to \mathcal{A}'$ with $d'(\underbrace{k, \vartheta}_{\text{failure scenario}}) = \hat{\pi} \in \mathcal{A}'$

Multiple CM case: $d' : \mathcal{T} \times [0,1]^r \times \mathcal{K} \to \mathcal{A}'$

$$\text{with } d'(k, \boldsymbol{\vartheta}, \boldsymbol{k}) = \hat{\pi} \in \mathcal{A}'$$

Loss function

under-estimation of p and π, resp., is more critical than over-estimation

propose asymmetric linear loss, i.e.

$$L(p, d(k) = \hat{p}) = \begin{cases} l_1(p - \hat{p}) & \text{if } \hat{p} \leq p \\ l_2(\hat{p} - p) & \text{if } \hat{p} > p \end{cases}$$

for the other cases: replace p and d by π and d', respectively.

Risk function

in the most general case of multiple CM we have

$$\mathcal{R}((\pi,\vartheta),d') = \sum_{k=0}^{n}\sum_{i=1}^{|\mathcal{K}|} L(\pi, d'(k,\vartheta,\boldsymbol{k}_i))$$

$$*\sum_{j=0}^{k}\xi_{ij}P(X'=j|\pi)$$

where $\xi_{ij} = P(K=k-j)$ with $K \sim GBi(k,\vartheta_{\boldsymbol{k}_i})$

$$i=1,\ldots,|\mathcal{K}|;\;\; j=0,\ldots,k$$

5. Bayes decisions and application of the CM failure model

consider only CM decision framework with a single CM

need to specify a prior $f(\pi)$

Bayes optimal solution minimizes the preposterior expected loss: with $\pi \sim Be(a, b)$ we obtain the preposterior distribution as Beta mixture

$$\pi | k, \vartheta \sim \sum_{j=0}^{k} \xi_j Be(a + j, b + n - j)$$

$\Rightarrow$ Bayes decision $\hat{\pi}^* = F^{-1}_{\pi|k,\vartheta}\left(\frac{l_1}{l_1 + l_2}\right)$

New approach

usual burn-in **strategy**: if failures occur, CM's need to be installed. Hereafter, the burn-in study has to be repeated.

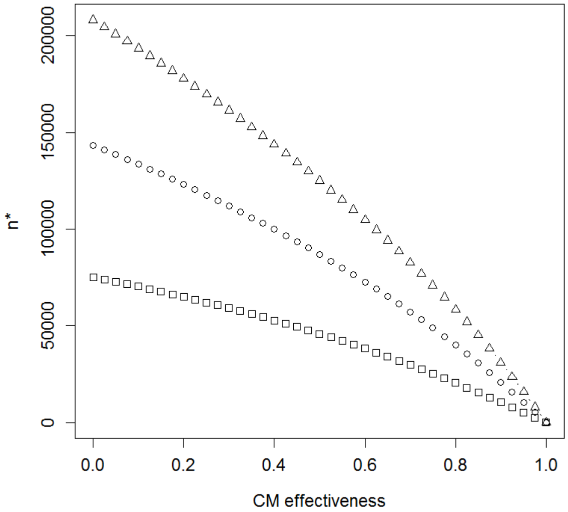
Our new approach: do not repeat burn-in, but extend the running burn-in study by increasing the sample size to $n' = n + n^*$ so that

$$\sum_{j=0}^{k} \xi_j P(X' \leq j | n', \hat{\pi}_{\text{target}} = \hat{p}_{\text{target}}) = 0.1$$

Rationale: Take $n^* < n$ additional devices and prove that the target $ppm-$level is still guaranteed on the basis of the CM model

Efficiency of the new approach: illustration for single CM case (different degrees of effectiveness) and $k = 1, 2, 3$.

Significant reduction of n^* for high effectiveness



6. Bayesian assessment of Weibull early life failure distributions

Burn-in settings (read-outs, burn-in time, ...) are typically assessed using a Weibull DFR distribution $Wb(a, b)$ with

$$\text{scale } a > 0 \text{ and shape } b \in (0, 1)$$

crucial point: joint prior $p(a, b)$

- There is no continuous conjugate joint prior
- Conjugate continuous-discrete joint prior: Gamma dist. for a, categorical distr. for b (Soland 1969)
- Jeffreys' prior: $p_J(a, b) \propto 1/ab$ (Sinha 1986)

We propose two alternatives:

- Histogram prior (specification remains still challenging)
- Dirichlet prior

Let $T \sim Wb(a, b)$ with density

$$f(t|a,b) \propto \begin{cases} t^{b-1}\exp(-(\frac{t}{a})^b) & t > 0 \\ 0 & \text{else} \end{cases}$$

where $a > 0, 0 < b < 1$

Burn-in read outs at fixed time points $t_1^*, \ldots, t_m^* > 0$

Specification of the prior:

$$F(t_i^*) \sim Be(u_i, v_i); u_i, v_i > 0, \quad i = 1, \ldots, m)$$

$u_i \quad \hat{=} \quad$ prior exp. number of early life failures before time t_i^*

$v_i \quad \hat{=} \quad$ prior exp. number of failures surviving burn-in time t_i^*

More efficiently, we summarize prior knowledge by means of a **Dirichlet prior**

$p_i = F(t_i^*) - F(t_{i-1}^*) =$ prob. of early failure within $(t_{i-1}^*, t_i^*]$

$\boldsymbol{p} = (p_1, \ldots, p_{m+1})^T \sim Dir(\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_{m+1}))$

Here we set $\vartheta_{m+1} = \vartheta^* - \sum_{i=1}^{m} \vartheta_i$

ϑ^* regulates prior confidence through

$$E(\boldsymbol{p}) = (\vartheta_1/\vartheta^*, \ldots, \vartheta_{m+1}/\vartheta^*)$$

Obviously: $\vartheta_i = u_i - u_{i-1};\ \ i = 1, \ldots, m+1$

$\Rightarrow$ complete specification: $\boldsymbol{p} \sim Dir(\boldsymbol{\vartheta})$ with $\boldsymbol{\vartheta} = \vartheta^* E(\boldsymbol{p})$

Joint prior $p(a, b)$ for Weibull parameters:

Draw samples of $p_1, \ldots, p_{m+1}$ and compute

$$F(t_i^*) = \sum_{j=1}^{i} p_j; \ \ i = 1, \ldots, m$$

Each pair $(F(t_i^*), F(t_j^*))$ with $i, j = 1, \ldots, m; i < j$ defines a sample (a_{ij}, b_{ij}) of the joint prior $p(a, b)$ via the equations

$$F(t_i^*) = 1 - \exp(-(t_i^*/a_{ij})^{b_{ij}}).$$
$$F(t_j^*) = 1 - \exp(-(t_j^*/a_{ij})^{b_{ij}}).$$

Explicitly, we get:

$$b_{ij} = \{\ln(-\ln(1 - F(t_j^*))) - \ln(-\ln(1 - F(t_i^*)))\}/\ln \frac{t_j^*}{t_i^*}$$

$$a_{ij} = \exp\{\ln t_i^* - \frac{1}{b_{ij}} \ln(-\ln(1 - F(t_i^*)))\}$$

For s Dirichlet draws $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_s$ we obtain $q * s$ pairs (a_{ij}, b_{ij})

where $q = \#\{(F(t_i^*), F(t_i^*)) : \quad i < j = 1, \ldots, m\}$

Updating Weibull early life failure distributions

Whenever failures occur, the current information on the Weibull lifetime distrib. should be updated.

Data might be available as

$$\boldsymbol{k} = (k_1, \ldots, k_{m+1})^T : k_i = \# failures \in (t^*_{i-1}, t^*_i]$$

or in form of time-to-failure data

$$\boldsymbol{t} = (t_1, \ldots, t_k)^T; \;\; k = \sum_{i=1}^{m+1} k_i$$

Notice: $k_{m+1} = \#$ failures not detected by the burn-in is not directly available.

Joint posterior $f(a, b|\boldsymbol{k})$

Regarding $\boldsymbol{k} = (k_1, \ldots, k_{m+1})$ as a sample from $MN(k, \boldsymbol{p})$, we obtain the posterior by

$$\text{sampling } (a_{ij}|\boldsymbol{k}, b_{ij}|\boldsymbol{k}); \quad i = 1, \ldots, s; \quad j = 1, \ldots q$$

according to the above equations using simulations from the **Dirichlet posterior**

$$\boldsymbol{p}|\boldsymbol{k} \sim Dir(\vartheta + \boldsymbol{k})$$

When we are given given time-to-failure data $\boldsymbol{t} = (t_1, \ldots, t_k)^T$, then the joint posterior $f(a, b|\boldsymbol{t})$ can be obtained according to the Metropolis-Hastings algorithm given in Kurz, Lewitschnig and Pilz (2013), where also HPD-regions for (a, b) are provided.

Update cont'd

Update of the Weibull lifetime distribution:

$$F(t|a,b) \longrightarrow F(t|\hat{a}^*,\hat{b}^*)$$

$$\begin{aligned} \text{where } (\hat{a}^*,\hat{b}^*) &= arg \max_{a>0,b<1} f(a,b|data) \\ &= \textbf{MAP estimate} \end{aligned}$$

Dynamical update through Bayesian learning

Example

3 standardized read-out times

$$t_1^* = 1h, t_2^* = 2h, t_3^* = 4h$$

read-outs based on Weibull early life failure distribution

$$T \sim Wb(a = 0.5, b = 0.75)$$

Dirichlet prior: $E(\boldsymbol{p}) = (0.81, 0.13, 0.05, 0.01)$

expected interval failure probabilities

setting $\vartheta^* = 100 \Rightarrow \boldsymbol{p} \sim Dir(81, 13, 5, 1)$

Dirichlet draws $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_s$ define samples

$$(F(t_1^*), F(t_2^*), F(t_3^*)); \quad i = 1, \ldots, s$$

We form pairs $(F(t_1^*), F(t_2^*))$ and $(F(t_1^*), F(t_3^*))$; and proceed as shown before to get

$$\hat{a}^* = 0.505, \hat{b}^* = 0.768$$

$\Rightarrow$ prior specification is suitable

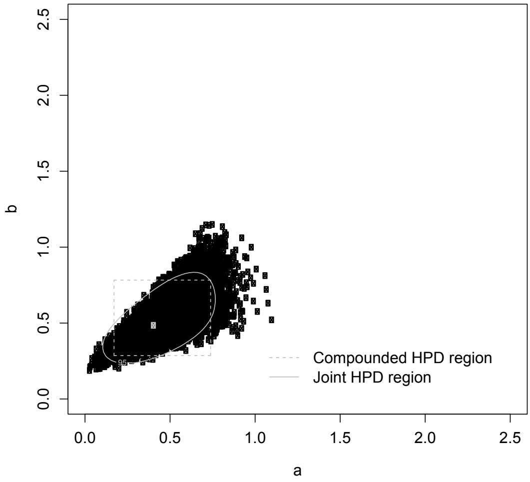
Data: $\boldsymbol{k} = (k_1 = 20, k_2 = 2, k_3 = 1, k_4 = 7)^T$

$k_4 = 7$ failures not detected within $t_3^* = 4$ hours
(burn-in time not adequate)

$\Rightarrow$ posterior: $\boldsymbol{p}|\boldsymbol{k} \sim Dir(101, 15, 6, 8)$,

$Wb(0.5, 0.75)$ shifted to $Wb(\hat{a}^* = 0.409, \hat{b}^* = 0.485)$

HPD region for Weibull parameters (a, b)



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