Statistical Modelling and Design for Quality Control and Reliability Analysis in Power Semiconductor Manufacturing Processes

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Semiconductor Manufacturing
involves > 100 process steps
silicon ingot \rightarrow slicing into sections
            flattening steps
      (lapping, polishing, cleaning, ...)
front-end processes back-end processes
            Final Chip
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Key issue in semiconductor manufacturing:

Reliability

especially in safety-critical applications Burn-in-study: most commonly applied failure screening technique.

Basis: bathtub curve describing hazard rate

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Early Life (decreasing λ)	Wear Out (increasing λ)/
Useful Life (constant λ)	

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Testing under accelerated stress conditions (increased temperature & voltage stress)

Burn-in: independently selected number of devices is investigated for early failures

Model for early failures: Weibull distribution Wb(a, b), b < 1.

Current ppm-requirement: 21ppm (Infineon)

Burn-in schemes different for logic and power devices. Here we focus on power devices.

Reasons for early failure: oxide particles, metallization defects.

Problem: only very few failures

 \Rightarrow it's impossible to efficiently fit a Weibull DFR distribution to burn-in data.

Way out: prove that early life failure probability $p \in$ target confidence area

Burn-in read-outs at discrete time points t_1, t_2, t_3

Report statistics: $k_j = \#$ failures in $(t_{j-1}, t_j]$

$$j = 1, 2, 3; t_0 = 0$$

Goal: *P* (early life failure after t_3 hours) $\leq 21 ppm$

Successful burn-in where $k = k_1 + k_2 + k_3 = 0$

(zero defect strategy)

Usually: Burn-in is re-started whenever a failure occurs

Current standard: introduction of countermeasures (CM) (ink out, design measures, optical inspection, ...) to reduce the failure probability p

Crucial: assessment of effectiveness of CM by experts.

Our aim:

- development of a statistical model for taking account of CM's
- avoid re-start of burn-in by planning additional number of items to be burnt for zero defects.

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n independently selected devices are stressed

$$X_i = \begin{cases} 0 & \text{if device } i \text{ passes the burn-in} \\ 1 & \text{if device } i \text{ fails within burn-in} \end{cases}$$

$$X = \sum_{i=1}^{n} X_i \sim Bi(n,p)$$

$$oldsymbol{x} = (x_1, \dots, x_n) \in \{0, 1\}^n; \hspace{0.2cm} k = oldsymbol{x}^T oldsymbol{x} \in \{0, 1, \dots, n\}$$
failures

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Clopper-Pearson interval estimation

$$I_{CP} = (\hat{p}_l, \hat{p}_u)$$
 where
 $P(X \ge k | \hat{p}_l) = lpha / 2$ and
 $P(X \le k | \hat{p}_u) = lpha / 2$

To obtain \hat{p}_l and \hat{p}_u , we use the well-known relationship with the Beta distribution

$$\hat{p}_l = F_{Z_l}^{-1}(\alpha/2)$$
 with $Z_l \sim Be(k, n-k+1)$
 $\hat{p}_u = F_{Z_u}^{-1}(1-\alpha/2)$ with $Z_u \sim Be(k+1, n-k)$

90% one-sided interval $I_p = [0, \hat{p}_u]; \alpha = 0.1$

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In a Bayesian framework, this relationship comes in naturally observing that the conjugate prior for p is the Beta distribution:

 $p \sim Be(a, b); a, b > 0$

$$\Rightarrow f(\boldsymbol{\rho}|\boldsymbol{x}) \propto l(\boldsymbol{\rho};\boldsymbol{x})f(\boldsymbol{\rho}) = \boldsymbol{\rho}^{a+k-1}(1-\boldsymbol{\rho})^{b+n-k-1}$$

i.e.
$$p|\mathbf{x} \sim Be(a^* = a + k, b^* = b + n - k)$$

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Bayesian equal-tail credible interval

$$C_e = (\hat{p}_l^*, \hat{p}_u^*)$$
 where $\hat{p}_l^* = F_{\rho|_{\boldsymbol{X}}}^{-1}(\alpha/2), \hat{p}_u^* = F_{\rho|_{\boldsymbol{X}}}^{-1}(1 - \alpha/2)$

Jeffreys' prior: a = b = 1/2

Choosing a = 1, b = 0 we have

$$p|\mathbf{x} = Be(k+1, n-k)$$

$$\hat{p}_u^* = \hat{p}_u$$

coincidence of one-sided Bayesian interval with Clopper-Pearson interval

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Repair is impossible for semiconductor devices; they either pass or fail within the burn-in. If a burn-in related failure occurs, then a CM is introduced (optical inspection, process improvement, ...) aiming to reduce p to $\pi \le p$.

Experts assess the CM's effectiveness $\vartheta \in [0, 1]$ = probability for correcting the failure.

Consider *k* failures for which a single CM with effectiveness $\vartheta \in [0, 1]$ is implemented in the process

Interpretation: There is a likelihood ξ_j that $j \le k$ failures would have occured or, equiv., k - j failures would have been solved (corrected) if the CM would have already been introduced before the burn-in study.

Let $K_l = \begin{cases} 1 & \text{failure } l \text{ solved} \\ 0 & \text{else} \end{cases}$

Clearly:
$$K = \sum_{l=1}^{k} K_l \sim Bi(k, \vartheta)$$

unknown number of failures that would have been caught by the CM

$$\Rightarrow (*)\xi_j = \mathcal{P}(\mathcal{K} = k - j); \ j \in \{0, \dots, k\}$$

Clopper-Pearson model for single CM

after the CM: $X' \sim Bi(n, \pi)$

Weighting of Clopper-Pearson upper limits according to (*) leads to assessing $\hat{\pi}$ as

$$\sum_{j=0}^{k} \xi_j \boldsymbol{P}(\boldsymbol{X}' \leq j | \hat{\pi}) = \alpha$$

Equivalently: using $P(X' \le j | \pi) = 1 - P(Z_j < \pi)$

with
$$Z_j \sim Be(j + 1, n - j); \ j = 0, ..., k$$

 $\Rightarrow \hat{\pi} = F_{Z'}^{-1}(1 - \alpha)$ quantile of

$$Z' \sim \sum_{j=0}^{k} \xi_j Be(j+1, n-j)$$
 Beta mixture

Multiple CM failure model

now consider $r \ge 1$ different CM's and denote $\vartheta = (\vartheta_1, \ldots, \vartheta_r) =$ vector of effectivenesses; $r \le k$

$$\boldsymbol{k} = (k_1 \dots, k_r); k_i = \#$$
 failures tackled by CM_i

with
$$\sum_{i=1}^{r} k_i = k$$

Now: $K = \sum_{l=1}^{k} K_l \sim GBi(k, \vartheta_k)$ generalized binomial, where $\vartheta_k = (\underbrace{\vartheta_1, \ldots, \vartheta_1}_{k_1 \text{ times}}, \underbrace{\vartheta_2, \ldots, \vartheta_2}_{k_2 \text{ times}}, \ldots, \underbrace{\vartheta_r, \ldots, \vartheta_r}_{k_r \text{ times}})$

We have developed an **efficient method** for computing generalized binomial probabilities employing sequential convolution.

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So far: ϑ_i ; $i = 1, \ldots, r$; were fixed

often: process experts are uncertain about the effectivenesses of the applied CM's.

a Beta-Binomial model for a single uncertain effectiveness, r = 1

$$\begin{split} \vartheta &\sim Be(u, v) \\ \mathcal{K} | \vartheta &\sim Bi(k, \vartheta) \\ &\Rightarrow \xi_j \quad = P(\mathcal{K} = k - j) = \int_0^1 P(\mathcal{K} = k - j | \vartheta) f(\vartheta) d\vartheta \\ &= \left(\begin{array}{c} k \\ k - j \end{array} \right) \frac{\Gamma(u + k - j)\Gamma(v + j)}{\Gamma(u + k + v)} \frac{\Gamma(u + v)}{\Gamma(u)\Gamma(v)} \end{split}$$

 $K \sim BeBi(k, u, v)$ Beta-Binomial

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Generalized Beta-Binomial model for more than a single uncertain effectiveness $K | \vartheta \sim GBi(k, \vartheta_k)$

$$\vartheta_i \sim Be(u_i, v_i); i = 1, \ldots, r$$

$$\Rightarrow P(K = k - j) = \int_{[0,1]^r} P(K = k - j | \vartheta) f(\vartheta) d\vartheta$$

 $K \sim GBeBi(k, u_1, \dots, u_r, v_1, \dots, v_r, k_1, \dots, k_r)$ no closed form solution, MC-integration

Take $n^* < n$ (even $n^* \ll n$) additional devices and prove that the target ppm-level is still guaranteed on the basis of the CM failure probability model

Efficiency of the new approach: illustration for single CM case (different degrees of effectiveness) and k = 1, 2, 3.

Significant reduction of n^* in case of high CM effectiveness!

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Required inspections in addition to n = 110000 for reaching target $p \le 21$ ppm at 90% CL with implemented CM(s) and k = 3, 2, 1 fails.

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Burn-in settings (read-outs, burn-in time, ...) are typically assessed using a Weibull DFR distribution Wb(a, b) with

scale a > 0 and shape $b \in (0, 1)$

crucial point: joint prior p(a, b)

- There is no continuous conjugate joint prior
- Conjugate continuous-discrete joint prior: Gamma dist. for *a*, categorical distr. for *b* (Soland 1969)
- Jeffreys' prior: $p_J(a, b) \propto 1/ab$ (Sinha 1986)

We propose two alternatives:

- Histogram prior (remains still challenging)
- Dirichlet prior
- Let $T \sim Wb(a, b)$ with density

$$f(t|a,b) \propto \left\{ egin{array}{cc} t^{b-1} \exp(-(rac{t}{a})^b) & t>0 \ & 0 & ext{else} \end{array}
ight.$$

with *a* > 0, 0 < *b* < 1

Burn-in read outs at fixed time points $t_1^*, \ldots, t_m^* > 0$

Data: $\mathbf{k} = (k_1 = 20, k_2 = 2, k_3 = 1, k_4 = 7)^T$

 $k_4 = 7$ failures not detected within $t_3^* = 4$ hours (burn-in time not adequate)

 \Rightarrow posterior: $\boldsymbol{p}|\boldsymbol{k} \sim Dir(101, 15, 6, 8)$

and Wb(0.5, 0.75) shifted to $Wb(\hat{a}^* = 0.409, \hat{b}^* = 0.485)$



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Stress testing in semiconductor processing for thin wafers (thickness $\leq 40 \mu \textit{m}$)

Kriging metamodel for stress prediction validated against Ramann spectroscopy measurements, FEM simulations

+ Modelling of electrical parameters (signals)



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Experiments

- physical experiments
- Computer experiments (Computer-based simulations like FEM)

Which simulations to run?

Main difference: Computer Models are **deterministic** Modification of classical DOE \Rightarrow DOCE

Math. model: $y = \tilde{f}(x_1 \dots, x_k)$, e.g. solution of ODE/PDE system $\mathbf{x} = (x_1, \dots, x_k)^T \in \mathcal{X}$ = experimental domain

replaced by meta-model

$$\mathbb{E} Y(\mathbf{x}) = f(x_1, \dots, x_k), f$$
 "close" to \tilde{f}

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Requirements for good designs:

- space filling property
- projective property
- computational efficiency

Compromise: LHD= Latin Hypercube Designs

w.l.o.g. experimental domain $\mathcal{X} = [0, 1]^k$

Designs: $d_n = (\mathbf{x}_1, \dots, \mathbf{x}_n) \in \mathcal{X}^n$, *n* runs, *k* factors



Figure: Regular (left) and latin hypercube design (right)



Figure: Maximin (left) and Minimax (right) designs

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0.0 0.4 0.8 0.0 0.4 0.8 Е u 2 <u>°°</u> 00 epsilon 60 0 0, 8 00 8 °° 80 5 h 40 8 0.8 0.8 0.4 0.8 0.0 0.4 0.0 0.4

Start design

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Optimal design for outeri=500



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Classical approach: Regression (response surface) modelling \Rightarrow prediction reduces to interpolation problem

e.g. quadratic RSM

$$y(\mathbf{x}) = \beta_0 + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i< j}^k \sum_{j=1}^k \beta_{ij} x_i x_j$$

For complex responses, LSE $\hat{\beta} = (X^T X)^{-1} X^T \mathbf{y}_d$

yields bad interpolations

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Remedy

Kriging models

 \Rightarrow response = realization of stochastic process

$$Y(\mathbf{x}) = \underbrace{\mu(\mathbf{x})}_{} + \underbrace{Z(\mathbf{x})}_{}$$

trend Gaussian Process (zero mean)

Effect: good approx. over a wide range of different designs and sample sizes and well-defined basis for statistical framework

$$Y(\cdot) \sim GP(\mu(\mathbf{x}), \sigma^2 R(\cdot, \cdot))$$

Main difference to geostatistical settings:

- x is no spatial coordinate vector
- usually, higher dimensional settings: K > 3

Covariance structure

covariance function:
$$Cov(Z(\mathbf{x}_i), Z(\mathbf{x}_j)) = \sigma^2 \underbrace{R(\mathbf{x}_i, \mathbf{x}_j)}_{\text{correlation function}}$$

Common assumptions:

covariance-stationarity, i.e.

$$R(\mathbf{x}_i,\mathbf{x}_j)=R(\mathbf{x}_i-\mathbf{x}_j)$$

(tensor-)product correlation structure

$$R(\mathbf{x}_i, \mathbf{x}_j) = \prod_{m=1}^{k} \underbrace{R_m(|x_{im} - x_{jm}|)}_{\text{univariate Matérn c.f.}}$$

Matérn c.f. $\nu = \frac{5}{2}$

$$R(d) = \left(1 - \frac{\tau^2}{\sigma^2}\right) * \left(1 + \frac{\sqrt{5}d}{\theta} + \frac{5d^2}{3\theta^2}\right) \exp\left(-\frac{\sqrt{5}d}{\theta}\right), \ d > 0$$



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MLE: available for β and σ^2 $\hat{\beta} = (X^T R_n^{-1}(\theta) X)^{-1} X^T R_n^{-1}(\theta) \mathbf{y}_d$ $\hat{\sigma}^2 = \frac{1}{n} (\mathbf{y}_d - X\hat{\beta})^T R_n^{-1}(\theta) (\mathbf{y}_d - X\hat{\beta})$ $\hat{\theta} = (\hat{\theta}_1, \dots, \hat{\theta}_k)^T$ Gauss-Newton (or genetic optimiz.) Optimal prediction:

$$\hat{Y}(\mathbf{x}^*) = \mathbf{f}(x^*)^T \hat{\boldsymbol{\beta}} + \mathbf{r}_0^T R_n^{-1} (\mathbf{y}_d - X \hat{\boldsymbol{\beta}}) \\ = \text{GLSE} + \text{smoothed residual}$$

where $\mathbf{r}_0^T = (R(\mathbf{x}_0 - \mathbf{x}_1), \dots, R(\mathbf{x}_0 - \mathbf{x}_n)), R_n = \text{correl. matrix}$

Implementation in R: DiceKriging

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Aims

- higher flexibility in meta-modelling
- numerical stability: robustness of parameter estimates, esp. for correlation parameters

Solution: Bayesian approach using additive models and (objective) reference priors

Side effect: high-dimensional optimization problems reduced to a few sub-routines of \leq 3 dimensions

Additive model:

$$\mathbb{E}Y(\mathbf{x}) = f_0 + \sum_{i=1}^{\kappa} f_i(x_i) + \sum_{i < j} f_{ij}(x_i, x_j) + \ldots + f_{12\ldots k}(x_{1,\ldots}, x_k)$$

Functional ANOVA Representation

Special case: first order GAM

$$\mathbb{E} Y(x_1,\ldots,x_k) = f_0 + \sum_{i=1}^k f_i(x_i)$$

 $f_1, ..., f_k$: smooth basis functions

- \Rightarrow non-parametric modelling of main effects
- Goal: Extension of classical GAM regression

Fo a good overview of the advantages of additive structures compared to fully parametric GP models in high dimensions see Dourante, Ginsbourger, Roustant (2012) **Novelty** of our recently proposed concept: Combination of AGP with robust reference priors proposed by Gu, Wang and Berger (submitted to AS 2017) + new sampling design scheme

Our new model: Second order Kriging AGP with

 $f_i \sim N(\mu_i, \sigma^2 R_i)$

$$f_{ij} \sim N(\mu_{ij}, \sigma^2 R_i R_j)$$

Result: AGP $Y(\mathbf{x}) \sim N(\mu, \sigma^2 R(\cdot, \cdot))$, locally constant trend

and
$$R(\mathbf{x}, \mathbf{x}') = \sum_{i=1}^{k} R_i(x_i, x_i') + \sum_{i=1}^{k} \sum_{j=i+1}^{k} R_i(x_i, x_i') R_j(x_j, x_j') + \delta_{\mathbf{xx}'} \tau^2$$

Renormalization such that

 $f^* := \frac{1}{\sqrt{m}} f \sim N\left(\frac{1}{\sqrt{m}} \mu, \sigma^2 R^*\right)$ $R^* := \frac{1}{m}R$ valid correlation matrix, m = # correlation components

Each component function has parameters $\Theta_i = (\mu, \sigma^2, \theta_i, \tau_i^2)$ for 1st order terms $\Theta_{ii} = (\mu, \sigma^2, \theta_i, \theta_i, \tau_{ii}^2)$ for 2nd order terms

Profile likelihood approach often fails, results in estimates $\hat{\theta}$ for which

(*)
$$R \approx I_n$$
 or $R \approx \mathbf{1}_n \mathbf{1}_n^T$
 $(\hat{\theta} \approx \mathbf{0})$ singular corr.m.
 \downarrow
bad ("impulse") prediction

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Remedy: robust Bayes prediction using reference priors of the form

 $\pi^{R}(\mu, \sigma^{2}, \theta^{*}) = \frac{\pi^{R}(\theta^{*})}{\sigma^{2}}$ \downarrow correl. parameters
where $\pi^{R}(\theta^{*}) \propto (\det I_{F}(\theta^{*}))^{1/2}$ \downarrow exp. Fisher information

Explicit representations for $I_F(\theta^*)$ available in Kazianka & Pilz (2012)

Result: proper posteriors $p(\theta^*|\mathbf{y}_d)$

Simplified estimate:
$$\hat{\theta}^* = arg \max_{\theta^*} p(\theta^* | \mathbf{y}_d)$$

posterior mode (to avoid MCMC)

Bayes Prediction

Bayes predictor of $Y(\mathbf{x}_0)$ for untried input \mathbf{x}_0 is based on the predictive distribution

$$p(Y_0|\mathbf{y}_d) = \int \underbrace{p(Y_0|\mathbf{y}_d, \theta^*)}_{Student-t} p(\theta^*|\mathbf{y}_d) d\theta^*$$

Simplification: Use plug-in predictor

$$\begin{split} \mu^* &:= E(Y_0|\mathbf{y}_d, \hat{\theta}^*) \\ &= \hat{\mu} + \mathbf{r}_0^T R_{\hat{\theta}^*}^{-1} (\mathbf{y}_d - \hat{\mu} \mathbf{1}_n) \\ \end{split} \\ \text{where } \hat{\mu} &= (\mathbf{1}_n^T R_{\hat{\theta}^*}^{-1} \mathbf{1}_n)^{-1} \mathbf{1}_n^T R_{\hat{\theta}^*}^{-1} \mathbf{y}_d \text{ GLSE} \end{split}$$

R-implementation fully described in Vollert, Ortner & Pilz (2019) is based on an iterative estimation scheme, using reparametrizations

Note: increasing nugget with increasing dimension k of input space

Due to additive structure, space-filling is important (for all variable projections)

Need **compromise** between LHD and regular grid designs: **Cut-FD** combines *HDMR Designs* based on a cut-center with *Factorial Designs*

 \Rightarrow $n_0 = 2^k + 2k$ boundary points +1 cut-point

< $10 \cdot k = n^*$ (recommended min.size for DOCE)

whenever $k \le 5$

Add $(n^* - n_0)$ points along (*ij*)-planes of cut point **x**_c

For k > 5 we recommend to use *Fractional Factorial Designs* instead of Full Factorials.

Example

3 commonly used test functions

Pepelyshev function: $x_i \in [0, 1]$; i = 1, 2, 3

 $f_1(\mathbf{x}) = 4(x_1 - 2 + 8x_2 - 8x_2^2)^2 + (3 - 4x_2)^2 + 16\sqrt{x_3 + 1}(2x_3 - 1)^2$

 $n_1 = 31$ samples

Park function: $x_i \in [0, 1]; i = 1, ..., 4$ $f_2(\mathbf{x}) = \frac{2}{3} \exp(x_1 + x_2) - x_4 \sin(x_3) + x_3$ $n_2 = 41$ samples

Friedman function: $x_i \in [0, 1]; i = 1, ..., 5$ $f_3(\mathbf{x}) = 10 \sin(x_1 x_2) + 20(x_3 - 0.5)^2 + 10x_4 + 5x_5$ $n_3 = 47$ samples



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All calculations in R,

Ihs package for constructing a maximin LHD, **DiceKriging** package for constructing GP models

Setup:

- Matérn correlation with $\nu = 5/2$ for all components
- comparison for 3 designs: random LHD, maximin LHD and Cut-FD

Criterion for comparisons: MAPE = mean absolute prediction error, measured (in %) at 25000 positions (generated by 50 random designs each containing 500 points)

- Cut-FD can better determine the actual structure of the test functions than maximin and other LHDs (found exact set of components for Pepelyshev and Friedman functions, maximin did not)
- Maximin LHD design was best with regard to MAPE (pred. power): For 5D-Friedman function MAPE < 4% (based on only n₃ = 47 sample points!)
- Solution Section 3 Control Control
- Simple random LHDs are least appropriate for approximation, getting even worse with increasing dimension

Results



Friedman

Designs

CutFD

- Maximin LHD
- Random LHD
- + DiceKriging

Work in progress:

AGP modelling for real DOCE applications based on FEMs for geometric and material parameter optimization problems, e.g. Magnetic field shaping for position and orientation detection systems



Model Performance based on random LHD

Component functions chosen by our algorithm:

- for B_x : f_s , f_b , f_{cs} , f_{as}
- for B_z : f_s , f_a , f_b , f_c , f_{as} , f_{bM} , f_{ab} , f_{bs} , f_{cs}



HORIZON 2020 & ECSEL Joint Undertaking Project "iDev40": Integrated Development 4.0 for Semiconductor Manufacturing

Project start: May 2018, End date: April 2021

Our project part will focus on Statistical Root-Cause Failure Analysis and Advanced Statistical and Bayesian Deep Learning Methodology. The project includes industrial partners Infineon Austria/Germany/Rom., AVL, ELMOS, Yazzoom, ETC, IBERMATICA,

... and guite a few European university research partners

Spatiotemporal Random Field Modeling and Bayesian Deep Learning Methods for Signal/Image Processing of wafer maps



Extension of AGP to Nearest Neighbour AGP

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